Analyzing simulation experiments with common random number, part II

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ANALYZING SIMULATION EXPERIMENTS WITH COMMON RANDOM NUMBER, PART II: RAO'S APPROACH

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ANALYZING SIMULATION EXPERIMENTS WITH COMMON RANDOM NUMBERS, PART II: RAO'S APPROACH

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Abstract

This note uses Rao (1959)'s approach based on Hotelling's statistic, to analyze a linear regression model with correlated responses. Correlated responses arise if a simulation model uses common random numbers. Rao's approach is compared to Kleijnen (1988)'s approach. Rao's lack of fit test is superior, which is quantified by a Monte Carlo study. Confidence intervals for individual regression parameters differ only slightly, when using Rao's and Kleijnen's approach respectively.

1. Introduction

Recently Kleijnen (1988) discussed the analysis of simulation experiments with common random number; Table 1 is reproduced from Kleijnen (1988, p. 66). These simulation data are analyzed through a linear regression model $\mathbf{y} = X\beta + \mathbf{e}$ with a nondiagonal covariance matrix. In this note we reconsider the analysis of this model, appyling Rao (1959).

TABLE 1

Data of Simulation Experiment

Factor combination (effects) $(\beta_1 \ldots \beta_Q)$	Replicated responses (seed 1)	(seed 2)	...	(seed m)	Average response	Estimated (co)variances
$x_{11} \ldots x_{1Q}$	y_{11}	y_{12}	...	y_{1m}	$\bar{y}_1$	$\hat{\sigma}_1^2 \hat{\sigma}_{12} \ldots \hat{\sigma}_{1n}$
$x_{21} \ldots x_{2Q}$	y_{21}	y_{22}	...	y_{2m}	$\bar{y}_2$	$\hat{\sigma}_2^2 \ldots \hat{\sigma}_{2n}$
$x_{i1} \ldots x_{iQ}$	y_{i1}	y_{i2}	...	y_{im}	$\bar{y}_i$	$\hat{\sigma}_i^2 \ldots \hat{\sigma}_{in}$
$x_{n1} \ldots x_{nQ}$	y_{n1}	y_{n2}	...	y_{nm}	$\bar{y}_n$	$\hat{\sigma}_n^2$

Table 1 shows that there are m independent observations on the n-variate vector $\mathbf{y} = (y_1, \ldots, y_i, \ldots, y_n)'$ which yields the following unbiased estimators of $\sigma_{ii'} = \text{cov}(y_i, y_{i'})$:

$$\hat{\sigma}_{ii'} = \frac{\sum_{r=1}^{m} (y_{ir} - \bar{y}_i)(y_{i'r} - \bar{y}_{i'})}{m-1} \qquad (i, i' = 1, \ldots, n) \quad (m \geq 2) \; , \qquad (1.1)$$

where $\bar{y}_i = \sum_{r=1}^{m} y_{ir}/m$. We do not assume a specific pattern for the covariance matrix; all we assume is that the covariance matrix $\Omega_y = (\sigma_{ii'})$ is non-singular. Kleijnen (1988, p. 67) proposed two different point estimators for the Q effects $\beta = (\beta_j)$. The Ordinary Least Squares (OLS) estimator is

$$\hat{\beta} = (X'X)^{-1} X' \bar{y} \qquad (1.2)$$

where $X = (x_{ij})$ with $j = 1, \ldots, Q$, and $\bar{y} = (\bar{y}_i)$; also see Table 1. The Estimated Generalized Least Squares (EGLS) estimator is

$$\hat{\tilde{\beta}} = (X' \hat{\Omega}_y^{-1} X)^{-1} X' \hat{\Omega}_y^{-1} \bar{y} \; , \qquad (1.3)$$

where $\hat{\Omega}_y = (\hat{\sigma}_{ii'})$ is assumed to be non-singular. The corresponding estimated covariance matrices of the estimators for β are

$$\hat{\Omega}_{\hat{\beta}} = (X'X)^{-1}X'\hat{\Omega}_y X(X'X)^{-1}/m \tag{1.4}$$

and

$$\hat{\Omega}_{\hat{\tilde{\beta}}} \approx (X'\hat{\Omega}_y^{-1}X)^{-1}/m \tag{1.5}$$

where $\approx$ means that (1.5) holds only asymptotically ($m \geq 25$?; see Kleijnen, 1988, p. 68).

2. Kleijnen (1988)'s versus Rao (1959)'s validation test

Thirty years ago Rao considered a related problem, namely multivariate linear regression analysis. Unfortunately he did not use the symbols that have become standard in regression analysis. In the following paragraph we shall first give his formulas and then translate them into the symbols of Kleijnen (1988). (Readers interested in the resulting formula, can skip to equation 2.4.b).

Rao (1959, p. 49) considers the linear model

$$E(\mathbf{y}) = \mathbf{A}\tau \tag{2.1.a}$$

TABLE 2

Symbols in Rao (1959) and Kleijnen (1988) in order of appearance

Rao :	$\mathbf{A}$	τ	p	m	Λ	$\mathbf{y}$	e	$\mathbf{S}$	f	r	$\hat{\tau}$	i
Kleijnen:	X	β	n	Q	Ω_y	$\bar{y}$	m	$\hat{\Omega}_y$	m-1	n-Q	$\hat{\tilde{\beta}}$	j

where $y = (y_1,...,y_p)'$, A is the $p \times m$ matrix of coefficients, and τ is the vector of m regression parameters. Kleijnen (1988, p. 67) uses the model

$$E(y) = E(\bar{y}) = X\beta \tag{2.1.b}$$

Let x := y mean that Rao's symbol x becomes Kleijnen's symbol y. So $A := X$, $\tau := \beta$, $p := n$, $m := Q$; also see Tables 1 and 2. Rao (1959, p. 49) further assumes "The dispersion matrix of $y_1,...,y_p$ is $e^{-1}\Lambda$ where e is a known constant and Λ is unknown, but an estimator S of Λ is available and has Wishart's distribution based on f degrees of freedom independently of $y_1,...,y_p$. The matrix Λ is assumed to be non-singular". Analogously, Kleijnen (1988, p. 67) assumes that the covariance matrix of $y_1,...,y_n$ is Ω_y and the covariance matrix of the averages $\bar{y}_1,...,\bar{y}_n$ is $\Omega_{\bar{y}} = \frac{1}{m}\Omega_y$. So if $\Lambda := \Omega_y$ and $y := \bar{y}$ then $e := m$. To check this translation we note that Rao's equation (2.6)

$$\min e(y-A\tau)'\Lambda^{-1}(y-A\tau) \tag{2.2.a}$$

corresponds to Kleijnen's EGLS criterion

$$\min(\bar{y}-X\beta)\Omega_{\bar{y}}^{-1}(\bar{y}-X\beta) \ . \tag{2.2.b}$$

We note that Rao's example leading to his equation (1.2) is wrong: $e = 1/n$ should be $e = n$. We further have $S := \hat{\Omega}_y$ with degrees of freedom $f := m-1$. Next Rao (1959, p. 50) introduces "r equal to p minus the rank of A". So $r = p - \text{rank}(A) := n - \text{rank}(X)$; usually $r := n-Q$; for saturated designs we have, by definition, $r := 0$. To test the model specification Rao uses Hotelling's statistic T_r which according to his equations (2.4) and (2.9) equals

$$T_r = \frac{e}{f}\min(y-A\tau)'S^{-1}(y-A\tau) \ . \tag{2.3.a}$$

Using (2.3) we get in Kleijnen's symbols:

$$T_r = \frac{1}{(m-1)} (\bar{y}-X\hat{\tilde{\beta}})'\hat{\Omega}_y^{-1}(\bar{y}-X\hat{\tilde{\beta}}) \ . \tag{2.3.b}$$

Rao's equation (2.5) states that Hotelling's statistic T_r corresponds to the F statistic:

$$F_{r,f+1-r} = \frac{f+1-r}{r} T_r \tag{2.4.a}$$

or in Kleijnen's symbols, assuming for simplicity of notation that rank(X) = Q,

$$F_{n-Q,m-n+Q} = \frac{m-n+Q}{n-Q} \frac{1}{(m-1)} (\bar{y}-X\hat{\tilde{\beta}})'\hat{\Omega}_y^{-1}(\bar{y}-X\hat{\tilde{\beta}}) \tag{2.4.b}$$

We can interpret this equation as follows. A perfect fit $(\bar{y}=X\hat{\tilde{\beta}})$ means that $T_r = 0$ and hence $F = 0$ and we do not reject the specified model; high estimated variances $\hat{\sigma}_i^2$ mean that large residuals $(\bar{y}-X\hat{\tilde{\beta}})$ are accepted. Note that, if m goes to infinity then $F_{n-Q,m-n-Q}$ approaches χ^2_{n-Q}. The F test exists only if n > Q (non-saturated design).

The F statistic of (2.5.b) is a <u>generalization of the F test for lack of fit</u> in the classical experimental design literature, which assumes $\hat{\Omega}_y = \sigma^2 \mathbf{I}$. The F test compares the estimated residuals (lack of fit) to the pure estimated noise $\hat{\Omega}_y$. Kleijnen (1988, p. 71) proposes an alternative approach: estimate β from one set of simulation data; use this estimate to predict the simulation output for a new input combination; compare the predictor $\hat{y}_{n+1}$ to the observed simulation output y_{n+1}, using a t statistic. Kleijnen (1988, p. 71) states: "This test is simplest if we make $\hat{y}_{n+1}$ ... and y_{n+1} independent, i.e., if we use a new seed for y_{n+1}". However, such an approach excludes cross-validation: if n > Q then cross-validation means that we delete one factor combination i (i=1,...,n) and estimate β from the remaining n-1 combinations $(X_{-i}, \bar{y}_{-i}, \hat{\Omega}_{y(-i)})$; the estimate for β is used to compute the OLS predictor $\hat{y}_i(\hat{\beta}_{-i})$ and/or the EGLS predictor $\hat{y}_i(\hat{\tilde{\beta}}_{-i})$; this predictor is compared to the simulation response $\bar{y}_i$, using Studentization, i.e., dividing the prediction error $\hat{y}_i - \bar{y}_i$ by its standard deviation $(\widehat{\mathrm{var}}(\hat{y}_i-\bar{y}_i))^{\frac{1}{2}}$. If no common random numbers are used, this standard deviation is simple to derive; see Kleijnen (1983). If, however, common random numbers are used, then the simulation responses y_i and $y_{i'}$ are correlated, and the question arises how sensitive this t test is to this correlation. Before we try to answer that question, we consider another aspect of the regression analysis.

3. Kleijnen versus Rao's confidence intervals for β

Both Rao (1959, pp. 56-57) and Kleijnen (1987, p. 218) state that tests for individual parameters β_j are relevant, only after the specification of the regression model is tested (see § 2). To obtain confidence intervals for the parameters, Rao proceeds as follows (actually he considers the more general problem of a set of linear hypotheses).

Rao (1959, p. 52) introduces $C = (A'S^{-1}A)^{-1} := (X'\hat{\Omega}_y X)^{-1}$ which equals $m\hat{\Omega}_{\hat{\tilde{\beta}}}$ asymptotically; see (1.5). Rao (1959, p. 53) defines c_{ii} as the i-th diagonal element of C; hence $c_{ii} := m\ \hat{var}(\hat{\tilde{\beta}}_j)$ asymptotically. His eq. (4.4) gives the following confidence limits to τ_i:

$$\hat{\tau}_i \pm t\left[\frac{e(f-r)}{c_{ii}f(1+T_r)}\right]^{-\frac{1}{2}} \tag{3.1.a}$$

where t has f-r degrees of freedom; in our symbols this becomes

$$\hat{\tilde{\beta}}_j \pm t_v\left[\frac{(m-1-n+Q)}{\hat{var}(\hat{\tilde{\beta}}_j)(m-1)(1+T_r)}\right]^{-\frac{1}{2}} \tag{3.1.b}$$

where $v = m-1-n+Q$ and $\hat{var}(\hat{\tilde{\beta}}_j)$ is computed from (1.5). Note that as m approaches infinity (or $m\uparrow\infty$), the confidence interval lengths go to zero ($t_v\downarrow z$ whre z denotes the standard normal variable, $\hat{var}(\hat{\tilde{\beta}}_j)(m-1) \approx$ constant, $T_r\downarrow 0$). Rao (1959, p. 57) states that "C [in the example] is the least squares variance of the estimator"; actually we showed that C equals m times that variance! We can rewrite (3.1.b) as follows

$$\hat{\tilde{\beta}} \pm t_v\ \hat{\sigma}(\hat{\tilde{\beta}}_j)\left\{\frac{(m-1)(1+T_r)}{(m-1)-(n-Q)}\right\}^{\frac{1}{2}} \tag{3.1.c}$$

where $\hat{\sigma}(\hat{\tilde{\beta}}_j)$ denotes $\{\hat{\mathrm{var}}(\hat{\tilde{\beta}}_j)\}^{\frac{1}{2}}$. In a saturate design we have $n = Q$; in practice n will not exceed Q very much. So assuming $(n-Q) \downarrow 0$ yields

$$\hat{\tilde{\beta}} \pm t_v \, \hat{\sigma}(\hat{\tilde{\beta}}_j) \, (1+T_r)^{\frac{1}{2}} \tag{3.2}$$

Eq. (2.4.b) proved that $T_r \downarrow 0$ if the regression model fits adequately or if there are many replications. Kleijnen (1988, p. 68) proposes to use EGLS only if $m \geq 25$ so that $\hat{\sigma}(\hat{\tilde{\beta}}_j)$ may be computed from (1.5); if $m \geq 25$ then t_v may be replaced by the standard normal variable z:

$$\hat{\tilde{\beta}} \pm z \, \hat{\sigma}(\hat{\tilde{\beta}}_j) \ . \tag{3.3}$$

4. A Monte Carlo experiment

We use a Monte Carlo experiment to estimate the α (or type I) and β (or type II) error of Rao's and Kleijnen's validation tests. We investigate Kleijnen's test only for the OLS estimator $\hat{\beta}$; see (1.2) and (1.4). [One reason for this restriction is that for the EGLS estimator $\hat{\tilde{\beta}}$ we know only the asymptotic covariance matrix; see (1.5). We expect that Kleijnen's test using EGLS will have a smaller β error (more power) than Kleijnen's test using OLS.] We further test the Studentized error using a t_v statistic with $v = m-1$ degrees of freedom. [Kleijnen (1983, p. 139) uses the standard normal statistic: $z = t_\infty$. Obviously $z^\alpha < t_v^\alpha$ for $v < \infty$ so that usage of z^α leads to higher type I errors and consequently to lower β errors.]

To estimate the actual α and β errors we apply Rao's and Kleijnen's tests 100 times per "case"; a case is defined by X, β, Ω and m. The factor "100" means that an estimated $\hat{\alpha}$ is a binomial variable with standard error $\{\hat{\alpha}(1-\hat{\alpha})/100\}^{\frac{1}{2}}$; for example, if $\hat{\alpha} = 0.20$ (we take a nominal α of 0.20) then its standard error is 0.04. Likewise the estimated power $1 - \hat{\beta}$ has the standard error $\{\hat{\beta}(1-\hat{\beta})/100\}^{\frac{1}{2}}$; this error reaches its maximum value of 0.05 for $\hat{\beta} = 0.50$. To estimate β we must specify an alternative model (or hypothesis H_1). We limit the Monte Carlo experiment to two factors x_2 and x_3 besides the dummy variable $x_1 = 1$; so we have

$$H_0 : E(\bar{y}_i) = \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} \tag{4.1}$$

and we add a two-factor interaction β_4 to get

$$H_1 : E(\bar{y}_i) = \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i2} x_{i3} . \tag{4.2}$$

For **X** we select orthogonal and non-orthogonal matrices; see Table 3. (Note that the equations in the preceding sections all use X excluding the last column of **X** in Table 3, that is, Q = 3). Note that we select β_4 small relative to the "main effects" β_2 and β_3. We fix m = 10. The response variances $var(y_i) = \sigma_i^2 = \sigma_{ii}$ are small for some cases (for example, case 1) and large for other cases (for example, case 2). For simplicity's sake we assume that the use of common random numbers in simulation yields constant correlation coefficients: $\rho_{ii'} = \sigma_{ii'}/(\sigma_i \sigma_{i'}) = \rho$ (a constant ρ was also assumed by several other authors; see Kleijnen, 1988). We investigate the effects of increasing correlation coefficients (more effective common seeds). If $\rho = 0$ then we might adapt Rao's approach; if we use different seeds in simulation then we know that $\rho = 0$ and we might replace the estimates $\hat{\sigma}_{ii'}$ in (1.1) by $\hat{\sigma}_{ii'} = 0$ for $i \neq i'$. This variant is shown in Tables 4 and 5 in the columns 0'. All results for case 1 are correlated since they use the same seed; they are independent of case 2, etc. (so rows 1 and 2 in Tables 4 and 5 are independent of all other rows).

TABLE 3

Monte Carlo Inputs

case	$x_1\ x_2\ x_3\ (x_2x_3)$	$\beta_1\ \beta_2\ \beta_3\ \beta_4$	$\sigma_1^2 \ldots \sigma_n^2$
1)	$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{vmatrix}$	(1,10,10,1)	(1,1,1,1)
2)	same	(1,10,10,5)	(10,20,30,40)
3)	same	(1,10,20,1)	(1,1,1,1)
4)	same	(1,10,20,3)	(10,20,30,40)
5)	$\begin{vmatrix} 1 & -1 & 4 & -4 \\ 1 & 2 & -3 & -6 \\ 1 & -3 & 2 & -6 \\ 1 & 4 & -1 & -4 \\ 1 & 1 & 1 & 1 \end{vmatrix}$	(1,10,10,0.5)	(1,1,1,1,1)
6)	same	(1,10,10,3)	10.(1,2,3,4,5)
7)	same	(1,10,20,0.5)	(1,1,1,1,1)
8)	same	(1,10,20,5)	10.(1,2,3,4,5)
9)	$\begin{vmatrix} 1 & 1 & 6 & 6 \\ 1 & 2 & -5 & -10 \\ 1 & 3 & 4 & 12 \\ 1 & 4 & -3 & -12 \\ 1 & 5 & 2 & 10 \\ 1 & 6 & -1 & -6 \end{vmatrix}$	(1,10,10,0.25)	(1,1,1,1,1,1)
10)	same	(1,10,10,0.25)	10.(1,2,3,4,5,6)
11)	same	(1,10,20,0.25)	(1,1,1,1,1,1)
12)	same	(1,10,20,0.25)	10.(1,2,3,4,5,6)
13)	$\begin{vmatrix} 1 & 2 & -1 & -2 \\ 1 & 4 & -1 & -4 \\ 1 & 6 & 1 & 6 \\ 1 & 8 & 1 & 8 \\ 1 & 10 & -1 & -10 \\ 1 & 12 & -1 & -12 \end{vmatrix}$	(1,10,10,0.25)	(1,1,1,1,1,1)
14)	same	(1,10,10,0.5)	10.(1,2,3,4,5,6)
15)	same	(1,10,20,0.75)	(1,1,1,1,1,1)
16)	same	(1,10,20,1)	10.(1,2,3,4,5,6)

TABLE 4

The Estimated α Error: $\hat{\alpha}$ (nominal α is 0.20)

Case		0'	ρ = 0	0.1	0.3	0.5	0.7	0.9
1	Rao	0.20	0.22	0.22	0.22	0.25	0.24	0.20
	Kleijnen		0.01	0.01	0.01	0.00	0.00	0.00
2	Rao	0.10	0.14	0.13	0.15	0.13	0.11	0.12
	Kleijnen		0.03	0.03	0.01	0.00	0.00	0.00
3	Rao	0.15	0.16	0.16	0.17	0.18	0.18	0.19
	Kleijnen		0.03	0.01	0.00	0.00	0.00	0.00
4	Rao	0.17	0.21	0.21	0.21	0.20	0.19	0.19
	Kleijnen		0.02	0.01	0.01	0.00	0.00	0.00
5	Rao	0.14	0.23	0.22	0.21	0.21	0.21	0.20
	Kleijnen		0.11	0.09	0.04	0.01	0.00	0.00
6	Rao	0.06	0.13	0.12	0.15	0.16	0.15	0.18
	Kleijnen		0.03	0.02	0.04	0.03	0.00	0.00
7	Rao	0.17	0.17	0.17	0.23	0.24	0.23	0.21
	Kleijnen		0.11	0.08	0.04	0.00	0.00	0.00
8	Rao	0.11	0.14	0.13	0.15	0.16	0.16	0.17
	Kleijnen		0.08	0.05	0.04	0.00	0.00	0.00
9	Rao	0.09	0.19	0.20	0.19	0.17	0.17	0.15
	Kleijnen		0.13	0.12	0.06	0.04	0.01	0.00
10	Rao	0.08	0.19	0.17	0.16	0.16	0.16	0.18
	Kleijnen		0.09	0.09	0.08	0.04	0.02	0.00
11	Rao	0.05	0.19	0.18	0.19	0.18	0.19	0.18
	Kleijnen		0.08	0.07	0.05	0.04	0.00	0.00
12	Rao	0.10	0.18	0.18	0.18	0.16	0.16	0.14
	Kleijnen		0.09	0.09	0.06	0.03	0.00	0.00
13	Rao	0.11	0.26	0.27	0.28	0.29	0.28	0.24
	Kleijnen		0.16	0.13	0.07	0.03	0.00	0.00
14	Rao	0.06	0.19	0.17	0.17	0.17	0.16	0.16
	Kleijnen		0.15	0.11	0.07	0.04	0.00	0.00
15	Rao	0.14	0.20	0.22	0.25	0.26	0.25	0.27
	Kleijnen		0.17	0.17	0.09	0.05	0.00	0.00
16	Rao	0.08	0.22	0.21	0.21	0.20	0.21	0.25
	Kleijnen		0.10	0.08	0.06	0.02	0.00	0.00

Next we consider the results $\hat{\alpha}$ in Table 4. If Rao's derivations (including use of asymptotic results) are correct, then the following null-hypothesis (denoted by H_0^R to avoid confusion with H_0 in 4.1) holds:

$$H_0^R : E(\hat{\alpha}) = \alpha \tag{4.2.a}$$

and the alternative hypothesis is

$$H_1^R : E(\hat{\alpha}) \neq \alpha \ . \tag{4.2.b}$$

Kleijnen's approach uses the Bonferroni inequality so that we expect it to be conservative:

$$H_0^K : E(\hat{\alpha}) \leq \alpha \tag{4.3.a}$$

and

$$H_1^K : E(\hat{\alpha}) > \alpha \ . \tag{4.3.b}$$

We have 16 cases with independent seeds, each case yielding one binomial variable $\hat{\alpha}$ for a particular ρ. Under H_0^R and H_0^K respectively, we may add the 16 binomial variates to get a single binomial variate based on a sample size of 1600. The resulting binomial test per ρ value does not lead to rejection of H_0^R or H_0^K, except for our adaptation of Rao (column 0'); then $\hat{\alpha}$ is significantly low. In other words, Rao's method yields the correct α error; Kleijnen's method is conservative indeed.

Now we turn to the estimated power $1 - \hat{\beta}$ in Table 5. A casual inspection of the data suggests that Rao's power is higher. Indeed Wilcoxon's rank test for a bivariate sample $(\hat{\beta}^R, \hat{\beta}^K)$ shows that Rao's test has significantly higher power; our adaptation of Rao (column 0') has non-significantly lower power than Kleijnen's test.

TABLE 5

The Estimated Power $1 - \hat{\beta}$

Case		0'	$\rho = 0$	0.1	0.3	0.5	0.7	0.9
1	Rao	0.70	0.75	0.82	0.83	0.92	0.98	1.00
	Kleijnen		0.39	0.40	0.40	0.36	0.37	0.32
2	Rao	0.79	0.79	0.82	0.91	0.90	0.98	1.00
	Kleijnen		0.44	0.43	0.44	0.43	0.39	0.39
3	Rao	0.74	0.76	0.79	0.82	0.90	0.98	1.00
	Kleijnen		0.43	0.42	0.38	0.36	0.36	0.34
4	Rao	0.54	0.55	0.57	0.61	0.81	0.87	0.97
	Kleijnen		0.21	0.18	0.14	0.09	0.06	0.00
5	Rao	0.66	0.73	0.75	0.84	0.91	0.97	1.00
	Kleijnen		0.60	0.56	0.58	0.53	0.52	0.61
6	Rao	0.68	0.73	0.75	0.79	0.88	0.79	1.00
	Kleijnen		0.57	0.55	0.52	0.51	0.48	0.51
7	Rao	0.67	0.75	0.75	0.85	0.96	0.99	1.00
	Kleijnen		0.59	0.60	0.61	0.60	0.59	0.64
8	Rao	0.96	0.98	0.99	1.00	1.00	1.00	1.00
	Kleijnen		0.94	0.94	0.95	0.93	0.94	0.98
9	Rao	0.67	0.81	0.83	0.88	0.94	0.99	1.00
	Kleijnen		0.73	0.74	0.72	0.69	0.63	0.57
10	Rao	0.10	0.21	0.20	0.22	0.20	0.23	0.38
	Kleijnen		0.14	0.12	0.07	0.04	0.02	0.00
11	Rao	0.71	0.79	0.80	0.87	0.94	1.00	1.00
	Kleijnen		0.77	0.77	0.71	0.66	0.64	0.58
12	Rao	0.12	0.20	0.19	0.19	0.19	0.21	0.41
	Kleijnen		0.15	0.12	0.08	0.04	0.03	0.00
13	Rao	0.12	0.29	0.29	0.32	0.40	0.45	0.68
	Kleijnen		0.17	0.14	0.09	0.04	0.01	0.00
14	Rao	0.06	0.21	0.21	0.20	0.20	0.19	0.21
	Kleijnen		0.15	0.11	0.07	0.04	0.01	0.00
15	Rao	0.47	0.57	0.60	0.65	0.78	0.94	1.00
	Kleijnen		0.58	0.55	0.54	0.54	0.54	0.54
16	Rao	0.10	0.30	0.30	0.30	0.29	0.33	0.41
	Kleijnen		0.15	0.10	0.09	0.04	0.00	0.00

Summarizing so far, we see that Kleijnen's test devised for $\rho = 0$, is conservative ($E(\hat{\alpha})<\alpha$) and (consequently) has lower power.

What is the effect of ρ? Kleijnen's test was devised for $\rho = 0$ (independent seeds). Table 4 suggests that for $\rho \uparrow 1$ we have $E(\hat{\alpha}) \downarrow 0$. Indeed we may fit the following model to the six observations per case:

$$E(\hat{\alpha}_{\ell}) = \gamma_0 + \gamma_1 \rho_{\ell} \qquad (\ell = 1,\ldots,6) \tag{4.4}$$

and $\hat{\gamma}_1$ is negative in each of the 16 cases. To understand this effect of ρ on $\hat{\alpha}$ we consider an extreme case: $\rho = 1$. Then all observations $\bar{y}_i$ lie on a straight line ($\bar{y}_i=\beta_1+\beta_2 x_{i2}+\beta_3 x_{i3}$); deleting one observation i does not affect $\hat{\beta}$; hence $\hat{y}_i$ does not change (and remains equal to $\bar{y}_i$); and we do not reject the model. A smaller $\hat{\alpha}$ means lower power $1 - \hat{\beta}$; see Table 5. So we should not apply Kleijnen's method in case of common seeds.

What is the effect of ρ on Rao's test? We have already seen that Rao's type I error remains α (see Table 4). Table 5 suggests that increasing ρ increases the power. If $\rho = 0.9$ then the power is often 100%. Indeed fitting a model similar to (4.4) gives a positive slope ($\hat{\gamma}_1>0$) except for case 14.

5. Conclusions

If simulation experiments use common random numbers as inputs, then the simulation outputs y are correlated. To analyze the simulation data, OLS and EGLS can be applied; EGLS seems to require many replications; see Kleijnen (1988). When EGLS is applied, Rao (1959) provides a lack of fit test which is better than Kleijnen's cross-validation test. If the EGLS regression (meta)model is not rejected, then confidence intervals for individual effects β_j can be based on either Rao (1959) or Kleijnen (1988) which differ only slightly.

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